COMPLETE TRIGONOMETRY FORMULA BANK

PART 1: Foundations, Identities, ASTC & Allied Angles (Class 9 to 11)

▶ 1. Right-Triangle Basics & Reciprocal Relations (Class 9 & 10)

Basic Definitions

$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{O}{H} \iff \csc\theta = \frac{H}{O}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{A}{H} \iff \sec\theta = \frac{H}{A}$$

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{O}{A} \iff \cot\theta = \frac{A}{O}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

★ The 3 Fundamental Pythagorean Identities & Shortcuts

$$\mathbf{\sin^2\theta + \cos^2\theta = 1} \implies \sin^2\theta = 1-\cos^2\theta, \quad \cos^2\theta = 1-\sin^2\theta$$

$$\mathbf{1+\tan^2\theta = \sec^2\theta} \implies \sec^2\theta - \tan^2\theta = 1 \implies (\sec\theta - \tan\theta) = \frac{1}{\sec\theta+\tan\theta}$$

Trick: If $\sec\theta + \tan\theta = k \implies \sec\theta - \tan\theta = \frac{1}{k}$

$$\mathbf{1+\cot^2\theta = \csc^2\theta} \implies \csc^2\theta - \cot^2\theta = 1 \implies (\csc\theta - \cot\theta) = \frac{1}{\csc\theta+\cot\theta}$$

Trick: If $\csc\theta + \cot\theta = m \implies \csc\theta - \cot\theta = \frac{1}{m}$

▶ 2. Standard Angles Reference Table (0° to 90°)

Ratio	**0° (0)**	**30° ($\frac{\pi}{6}$)**	**45° ($\frac{\pi}{4}$)**	**60° ($\frac{\pi}{3}$)**	**90° ($\frac{\pi}{2}$)**	**Complementary ($90^\circ-\theta$)**
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sin(90^\circ-\theta)=\cos\theta$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	$\cos(90^\circ-\theta)=\sin\theta$
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	ND	$\tan(90^\circ-\theta)=\cot\theta$

Reciprocals: $\csc\theta = 1/\sin\theta$, $\sec\theta = 1/\cos\theta$, $\cot\theta = 1/\tan\theta$ *(ND = Not Defined / Infinite)*

▶ 3. ASTC Quadrant Map & Allied Angles Master Table (Class 11)

ASTC Quadrant Sign Matrix

Quadrant II (90° to 180°) [**S**]: $\sin, \csc$ **are (+)** $\cos, \sec, \tan, \cot < 0$	**Quadrant I** (0° to 90°) [**A**]: **ALL 6 are (+)** All positive
Quadrant III (180° to 270°) [**T**]: $\tan, \cot$ **are (+)** $\sin, \csc, \cos, \sec < 0$	**Quadrant IV** (270° to 360°) [**C**]: $\cos, \sec$ **are (+)** $\sin, \csc, \tan, \cot < 0$

Even / Odd Parity:

$\cos(-\theta) = \cos\theta$, $\sec(-\theta) = \sec\theta$ **(Even)**

$\sin(-\theta) = -\sin\theta$, $\tan(-\theta) = -\tan\theta$ **(Odd)**

Complete Allied Angles Quick-Reference Table

Angle (α)	**Quadrant**	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
$\mathbf{90^\circ}-\theta$	I (All +)	$+\cos\theta$	$+\sin\theta$	$+\cot\theta$
$\mathbf{90^\circ}+\theta$	II (sin +)	$+\cos\theta$	$-\sin\theta$	$-\cot\theta$
$\mathbf{180^\circ}-\theta$	II (sin +)	$+\sin\theta$	$-\cos\theta$	$-\tan\theta$
$\mathbf{180^\circ}+\theta$	III (tan +)	$-\sin\theta$	$-\cos\theta$	$+\tan\theta$
$\mathbf{270^\circ}-\theta$	III (tan +)	$-\cos\theta$	$-\sin\theta$	$+\cot\theta$
$\mathbf{270^\circ}+\theta$	IV (cos +)	$-\cos\theta$	$+\sin\theta$	$-\cot\theta$
$\mathbf{360^\circ}-\theta$	IV (cos +)	$-\sin\theta$	$+\cos\theta$	$-\tan\theta$

Golden Rule: At $180^\circ/360^\circ \implies$ ratio remains **SAME**; at $90^\circ/270^\circ \implies$ changes to **CO-FUNCTION**. Sign depends on original function.

▶ 4. Compound Angles (Addition & Subtraction)

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \qquad \sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \qquad \cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Sign Trap: In $\cos(A+B)$, the sign flips to **MINUS** $(-)$, and in $\cos(A-B)$ it flips to **PLUS** $(+)$!

▶ 5. Double (2θ), Sub-Multiple ($\theta/2$) & Triple (3θ) Angles

Double Angle Formulas (2θ)

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Triple Angle Formulas (3θ)

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

★ Golden $1 \pm \cos$ Forms (Class 12 Calculus Must-Know)

Standard 2θ Forms:

$$\mathbf{1} - \cos \mathbf{2}\theta = \mathbf{2}\sin^{\mathbf{2}}\theta \implies \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\mathbf{1} + \cos \mathbf{2}\theta = \mathbf{2}\cos^{\mathbf{2}}\theta \implies \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$$

Sub-Multiple ($\theta/2$) Calculus Forms:

$$\mathbf{1} - \cos\theta = \mathbf{2}\sin^{\mathbf{2}}\left(\frac{\theta}{\mathbf{2}}\right), \qquad \mathbf{1} + \cos\theta = \mathbf{2}\cos^{\mathbf{2}}\left(\frac{\theta}{\mathbf{2}}\right)$$

$$\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$$

$$\frac{1-\cos\theta}{\sin\theta} = \tan\left(\frac{\theta}{2}\right), \qquad \frac{1+\cos\theta}{\sin\theta} = \cot\left(\frac{\theta}{2}\right)$$

▶ 6. Transformation Formulas (Product $\longleftrightarrow$ Sum / Difference)

Products $\longrightarrow$ Sum / Difference (De-Factorization)

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sum / Difference $\longrightarrow$ Products (C-D Factorization)

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -\mathbf{2}\sin\left(\frac{\mathbf{C+D}}{\mathbf{2}}\right)\sin\left(\frac{\mathbf{C-D}}{\mathbf{2}}\right)$$

$$= \mathbf{2}\sin\left(\frac{\mathbf{C+D}}{\mathbf{2}}\right)\sin\left(\frac{\mathbf{D-C}}{\mathbf{2}}\right)$$

▶ 7. Principal Value Branches: Domain & Range (Must-Know for Boards & CET)

Function	Domain (x)	Principal Value Branch / Range (y)	Key Exam Note
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	Quadrants IV & I ($-\pi/2 \le y \le \pi/2$)
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$	Quadrants I & II ($0 \le y \le \pi$)
$y = \tan^{-1} x$	$\mathbb{R} = (-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	Open Interval (Asymptotes at $\pm\pi/2$)
$y = \csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	Excludes $y = 0$
$y = \sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	Excludes $y = \pi/2$
$y = \cot^{-1} x$	$\mathbb{R} = (-\infty, \infty)$	$(0, \pi)$	Open Interval ($0 < y < \pi$)

▶ 8. Fundamental Properties & Negative Angle Traps

Negative Argument Properties (Crucial!)

Type 1 (Odd-Like: Direct Minus):

$$\sin^{-1}(-x) = -\sin^{-1} x, \quad x \in [-1, 1]$$
$$\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in \mathbb{R}$$
$$\csc^{-1}(-x) = -\csc^{-1} x, \quad |x| \ge 1$$

Type 2 (Even-Like: $\pi-$ Angle Trap):

$$\cos^{-1}(-\mathbf{x}) = \pi - \cos^{-1} \mathbf{x}, \quad x \in [-1, 1]$$
$$\cot^{-1}(-\mathbf{x}) = \pi - \cot^{-1} \mathbf{x}, \quad x \in \mathbb{R}$$
$$\sec^{-1}(-\mathbf{x}) = \pi - \sec^{-1} \mathbf{x}, \quad |x| \ge 1$$

Complementary Pairs & Reciprocal Relations

Complementary Co-Function Pairs ($\pi/2$ Sum):

$$\sin^{-1} \mathbf{x} + \cos^{-1} \mathbf{x} = \frac{\pi}{\mathbf{2}}, \quad x \in [-1, 1]$$
$$\tan^{-1} \mathbf{x} + \cot^{-1} \mathbf{x} = \frac{\pi}{\mathbf{2}}, \quad x \in \mathbb{R}$$
$$\sec^{-1} \mathbf{x} + \csc^{-1} \mathbf{x} = \frac{\pi}{\mathbf{2}}, \quad |x| \ge 1$$

Reciprocal Conversions:

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right), \quad \sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right) \quad (|x| \ge 1)$$

$$\cot^{-1} x = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right), & x > 0 \\ \pi + \tan^{-1}\left(\frac{1}{x}\right), & x < 0 \end{cases}$$

▶ 9. Addition / Subtraction of Inverse Tangent & $2\tan^{-1} x$ Conversions

Sum & Difference Formulas for $\tan^{-1}$

If $xy < 1$ (Standard Case):

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$$

If $x > 0, y > 0$ and $xy > 1$ (Quadrant II Trap):

$$\tan^{-1} \mathbf{x} + \tan^{-1} \mathbf{y} = \pi + \tan^{-1}\left(\frac{\mathbf{x} + \mathbf{y}}{\mathbf{1} - \mathbf{xy}}\right)$$

Difference Formula ($xy > -1$):

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right)$$

★ High-Yield $2\tan^{-1} x$ Multiple Conversions

Single Most Tested in Differentiation & Integration:

$$\mathbf{2}\tan^{-1} \mathbf{x} = \sin^{-1}\left(\frac{\mathbf{2x}}{\mathbf{1} + \mathbf{x^2}}\right), \quad |x| \le 1$$
$$\mathbf{2}\tan^{-1} \mathbf{x} = \cos^{-1}\left(\frac{\mathbf{1} - \mathbf{x^2}}{\mathbf{1} + \mathbf{x^2}}\right), \quad x \ge 0$$
$$\mathbf{2}\tan^{-1} \mathbf{x} = \tan^{-1}\left(\frac{\mathbf{2x}}{\mathbf{1} - \mathbf{x^2}}\right), \quad -1 < x < 1$$

Self-Inverse Cancellation Rules:

$$\sin(\sin^{-1} x) = x \quad (\forall x \in [-1, 1])$$
$$\sin^{-1}(\sin\theta) = \theta \iff \theta \in [-\pi/2, \pi/2] \text{ (Must lie in PVB!)}$$